Contents

1 Basic Linear Algebra

$$0 := \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}$$

$$A = B \Leftrightarrow a_{ij} \in A = b_{ij} \in B$$

$$A + 0 = 0 + A = A \qquad A + (-A) = 0$$

$$A \cdot x = b \quad \Rightarrow \quad x = A^{-1} \cdot b$$

$$a_{ij} \in A \Leftrightarrow (-a_{ij}) \in (-A)$$

$$A+(B+C) = (A+B)+C \qquad A(BC) = (AB)C$$

$$(\lambda + \mu) \cdot A = \lambda \cdot A + \mu \cdot A$$

A has m rows and n columns $\Leftrightarrow A \in M(m \times n)$

A linear system of equations can be translated to matrizes directly.

$$\begin{array}{ccccccc} -3x & + & -3y & -3z & = & -3 \\ -2x & + & 2y & z & = & 0 \\ x & + & -3y & 3z & = & 0 \end{array}$$

$$\Rightarrow A = \left(\begin{array}{cccc} -3 & -3 & -3 & -3 \\ -2 & 2 & 1 & 0 \\ 1 & -3 & 3 & 0 \end{array} \right)$$

To create an extended coefficient matrix, you have to add the solutions as the most right column (thus $\{A, b\}$). If you want to solve this system, you have to find x in $A \times x = b$, where A are the coefficients of the system and b are the solutions.

This can be done by performing arithmetic row-wise operations. An example is taking the third row minus $\frac{1}{2}$ times the second row. The result is a tuple $(0, -4, 2.5, 0)$ which can replace . Each operation returns a factor (here: $-\frac{1}{2}$); we will need this for decompositions. Okay, the result can be taken as new second row. So how is the general algorithm?

1.1 Gaussian elimination

Input: Invertible square matrix
Output: triangular form
Wolframalpha: RowReduce[A]
Skriptum. page 9

In a $m \times n$ matrix (m rows, n columns), we want to reach the structure of an upper triangular matrix. So if the result of such an operation is $(0, -4, 2.5, 0)$, we will prefer to use it as the second row (because of the one zero to the left). We will perform operations until we reach the expected structure; the "triangular form" (numerical analysis) or "row echelon form" (abstract algebra).

$$\begin{pmatrix} -3 & -3 & -3 & -3 \\ 0 & -4 & 2.5 & 0 \\ 0 & 0 & -0.5 & -1 \end{pmatrix}$$

This structure can be easily transformed to the solution of the linear system.

$$-0.5z = -1$$

$$-4y + 2.5 \cdot (-1) = 0$$

$$y = \frac{5}{4}$$

$$-3x - \frac{15}{4} = \frac{12}{4}$$

$$x = -\frac{9}{4}$$

Otherwise we can continue the elimination algorithm to create an identity matrix on the left side (columns 1–3 here). This way we can read the variable values immediately. This algorithm is called Gauss-Jordan Elimination.

1.2 Notes

- Pivot elements are the most-left numbers of the rows. -3, -4 and -0.5 in the previous example.
- rank A is the number of rows with non-zero pivot elements in the triangular matrix of A. rank $A = 3$ in the previous example.
- Sometimes swapping rows is necessary to reach a triangular structure.
- If there are only non-zero pivots, there is only one solution for the linear system. Otherwise we don't know anything about solutions.
- There are 3 operations that can be performed (elementary row operations):
  - Swapping rows
  - multiplication of a row with $\lambda \neq 0$
  - addition of row i with row j
- If there is only one solution the equivalent equation ($A \cdot x = 0$) can only be solved by $x = 0$.
- The system $Ax = b$ has a solution if

$$\operatorname{rank}(A \mid b) = \operatorname{rank}(A)$$

- There is one unique solution if

$$\operatorname{rank}(A \mid b) = \operatorname{rank}(A) = n$$

- There is one unique solution in a quadratic system if $\det(A) \neq 0$.

1.3 Matrix multiplication

Input: Two matrices A and B where number of columns(A) = number of rows(B)
Output: One matrix C
Wolframalpha: $A * B$
Skriptum. page 5

$$A \cdot B = C$$

$$\begin{pmatrix} 2 & 3 & 6 \\ 4 & 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 50 & 66 \\ 35 & 56 \end{pmatrix}$$

50 is the sum of $2 \cdot 1 + 3 \cdot 2 + 6 \cdot 7$. Matrix multiplication is *not* commutative.

1.4 Arithmetic matrix operations

Matrix additions or operations with a scalar happen element-wise.

1.5 Set operations

See figure 1. Copyright Wikipedia.

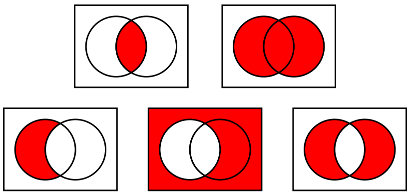

Figure 1: Basic set operations from left to right: first row: Intersection (∩), union (∪). second row: relative complement ($A \setminus B$), complement $U \setminus A$ (U is universal set), symmetric difference ($A \Delta B = (A \setminus B) \cup (B \setminus A)$)

1.6 Transposed matrix

Input: A matrix $A \in M(m \times n)$
Output: Matrix $A^T \in M(n \times m)$
Wolframalpha: Transpose[A]
Skriptum. page 6

$$\begin{pmatrix} 12 & 6 \\ 2 & 3 \\ 4 & 8 \end{pmatrix}^T = \begin{pmatrix} 12 & 2 & 4 \\ 6 & 3 & 8 \end{pmatrix}$$

$$(a_{ij})^T = (a_{ji})$$

$$(A+B)^T = A^T + B^T$$

$$(A \cdot B)^T = B^T \cdot A^T$$

$$A = A^T \Leftrightarrow A \text{ is a "symmetrical matrix"}$$

1.7 Inverse matrix

Input: $A \in M(n \times n), \text{rank } A = n$
Output: $C \in M(n \times n)$ if A is regular
Wolframalpha: A^{-1}
Skriptum. page 16

If A has an inverse matrix, A is called "regular matrix"; "singular" otherwise.

$$\begin{array}{lrcl} A \text{ is regular}: & (A \cdot B)^{-1} & = & B^{-1} \cdot A^{-1} \\ A^{-1} \text{ is regular}: & (A^{-1})^{-1} & = & A \\ A^T \text{ is regular}: & (A^T)^{-1} & = & (A^{-1})^T \end{array}$$

$$A \in M(n \times n) \quad \Rightarrow A \cdot A^{-1} = I$$

The inverse matrix of A is A^{-1}. This matrix can be found by solving the system:

$$(A, I) = \left(\begin{array}{ccc|cccc} a_{11} & \dots & a_{1n} & 1 & 0 & \dots & 0 \\ a_{21} & \dots & a_{2n} & 0 & 1 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} & 0 & 0 & \dots & 1 \end{array}\right)$$

Once there is an identity matrix on the left side (by elementary row-wise operations), the inverse matrix can be read from the right side of the separator.

2 Decompositions

LUP decomposition $PA = LU$

LDU decomposition $A = LDU$

LU decomposition with full pivoting
$PAQ = LU$

2.1 LU decomposition

Input: $A \in M(m \times n)$
Output: L and U with $A = LR$
Wolframalpha: LUDecomposition[]
Skriptum. page 18

$$A = L \cdot U \quad \text{(without swapped rows)}$$

- U is matrix A in triangular form
- L is a quadratic matrix with ones in the diagonal and below negative factors created by row-wise operations performed before.

$$A = P^T \cdot L \cdot R \quad \text{(with swapped rows)}$$

- U is matrix A in triangular form
- L is a quadratic matrix with ones in the diagonal and below negative factors created by row-wise operations performed before. While swapping rows, you have to swap all components of the rows left to the right ones accordingly.
- Permutation matrix P can be created by constructing an identity matrix and swapping all rows you did with L. $P^T = P^{-1}$.

A solution for $Ax = b$ can be found by applying

$$L \cdot y = b$$

$$R \cdot x = y$$

2.2 QR decomposition

Input: $A \in M(m \times n), m \geq n$ and linear independent column vectors
Output: $Q(m \times n)$ and $R(n \times n)$ with $A = QR$
Wolframalpha: QRDecomposition$[A]$
Skriptum. page 58

1. Apply Gram-Schmidt-Process to column vectors
2. $Q = \{q_1, q_2, \ldots\}$
3. $Q^T A = R \Rightarrow R$

$$a$$

Q is a matrix of orthonormal column vectors and R is an invertible upper triangular matrix.

3 Determinant

Input: $A \in M(n \times n)$
Output: a scalar
Wolframalpha: Det$[A]$
Skriptum. page 21

$$n = 1 : \det A = (a,) := a$$

$$n = 2 : \det A := \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} := a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

For $n = 3$, it's recommended to evaluate the determinant by using the rule of Sarrus. This can be done by adding as many column duplicates as necessary to get valid diagonals (simply $2m - 1$). This structure allows you to simply read all necessary addition and subtraction operations (see figure 2).

$$\det A = 1 \cdot 1 \cdot 1 + 2 \cdot 3 \cdot (-1) + (-1) \cdot 0 \cdot 2$$

$$-[(-1) \cdot 1 \cdot (-1)] - [1 \cdot 3 \cdot 2] - [2 \cdot 0 \cdot 1] = -12$$

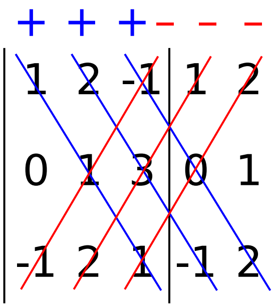

Figure 2: Rule of sarrus

The algebraic complement A'_{ij} of A is the determinant of the $(n-1) \times (n-1)$ matrix created by removing row i and column j.

$$A = (a_{ij}) \in M(n \times n)$$

$$\det A = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} A'_{ij}$$

- $\det A^T = \det A$
- Entire row or column is zero $\Rightarrow \det A = 0$
- $A \in M(n \times n), \lambda \in \mathbb{K} : \det(\lambda A) = \lambda^n \det A$
- Two rows or columns are identical: $\det A = 0$
- $\det(A \cdot B) = \det A \cdot \det B$
- $\det(A + B) \neq \det A + \det B$

A is regular $\Leftrightarrow \operatorname{rank} A = n \Leftrightarrow \det A \neq 0$

4 Vectors

Wolframalpha: $\{\{1\}, \{2\}, \{3\}\}$
Skriptum. page 24

$$\lambda, \mu \in \mathbb{R}$$

- Vector $\vec{v}$ with $\|\vec{v}\| = 0$ is called Null vector.
- Vector $\vec{v}$ with $\|\vec{v}\| = 1$ is called Unit vector.

- $\vec{a} + 0 = \vec{a}, \quad \vec{a} \cdot 0 = 0$
- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- $(\vec{a} + \vec{b}) + c = \vec{a} + (\vec{b} + c)$
- $\|\lambda \cdot \vec{a}\| = |\lambda| \cdot \|\vec{a}\|$
- $\lambda \cdot (\vec{a} + b) = \lambda \cdot \vec{a} + \lambda \cdot b$
- $(\lambda + \mu) \cdot \vec{a} = \lambda \cdot \vec{a} + \mu \cdot \vec{a}$
- $\|\vec{a} + \vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$

5 Norm (length) of a vector

Input: A vector $\vec{v}$
Output: A scalar
Wolframalpha: Norm[{2, 3}]

$$\|\vec{v}\| = \sqrt{\sum_i v_i^2}$$

6 Products of vectors

6.1 Dot product (of vectors)

Input: 2 arbitrary vectors $\vec{a}$ and $\vec{b}$
Output: A scalar
Wolframalpha: vector{1, 2, 3}. vector{2, 3, 4}
Skriptum. page 26
Deutsch "Skalarprodukt"

$$\langle a, b\rangle = \|a\| \cdot \|b\| \cdot \cos\varphi$$

- $\langle \vec{a}, \vec{b}\rangle = \langle \vec{b}, \vec{a}\rangle$
- $\langle \vec{a}, \vec{b} + \vec{c}\rangle = \langle \vec{b}, \vec{a}\rangle + \langle \vec{a}, \vec{c}\rangle$
- $\langle \vec{a}, \vec{b}\rangle = 0 \Leftrightarrow \vec{a} \perp \vec{b}$
- $\langle \vec{a}, \vec{a}\rangle = \|\vec{a}\|^2$
- $\vec{a} \in \mathbb{R}^3 : \langle \vec{a}, \vec{b}\rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$
- $\vec{a} \in \mathbb{R}^3 : \|\vec{a}\| = \sqrt{\langle \vec{a}, \vec{a}\rangle} = \sqrt{a_1^2 + a_2^2 + a_3^3}$

6.2 Cross product

Input: 2 arbitrary vectors $\vec{a}$ and $\vec{b}$
Output: A scalar c
Wolframalpha: {1, 2, 3} cross{2, 3, 4}
Skriptum. page 27
Deutsch "Vektorprodukt"

$$\vec{a}, \vec{b} \in \mathbb{R}^3 : \quad c := \vec{a} \times \vec{b}$$
$$c := \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin\varphi$$

Example:

$$\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \cdot 1 - 5 \cdot 6 \\ -(2 \cdot 1 - 5 \cdot 3) \\ 2 \cdot 6 - 4 \cdot 3 \end{pmatrix} = \begin{pmatrix} -26 \\ 13 \\ 0 \end{pmatrix}$$

6.3 Triple product

Input: 3 arbitrary vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$
Output: A scalar
Wolframalpha: {1, 2, 3} cross{2, 3, 4}
Skriptum. page 27
Deutsch "Spatprodukt"

$$\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$$
$$|\langle \vec{a} \times \vec{b}, \vec{c}\rangle| = \|\vec{a} \times \vec{b}\| \cdot \|\vec{c}\| \cdot \cos\angle(\vec{a} \times \vec{b}, \vec{c})$$

7 Linear maps

- injective (injections can be undone)

  $$\forall x_1, x_2 \in A : f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$$

- surjective (each element has a root):

  $$\forall y \in B : \exists x \in A : f(x) = y$$

- bijective = injective and surjective

8 Vector spaces

$\mathbb{K}$ is either $\mathbb{R}$ or $\mathbb{C}$. $\mathbb{P}_m$ is vector space of polynomials with degree m at maximum. A non-empty set V is called vector space of $\mathbb{K}$ if

1. The sum of $a, b \in V$ is defined and in V

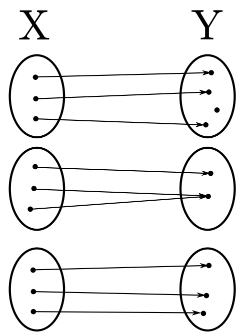

Figure 3: Relations of sets (top to bottom) 1. injective 2. surjective 3. bijective

2. The product of $\lambda \in \mathbb{K}$ and $a \in V$ $(= \lambda \cdot a)$ is defined and an element of V

- null vector: $0 \in V, \forall a \in V : a + 0 = a$
- unit vector: $1 \in V, \forall a \in V : a \cdot 1 = a$
- negative vector: $(-a) \in V, a + (-a) = 0$
- Is V a vector space of K. A non-empty subspace $U \subset V$ is called "linear subspace of V" if $\lambda \in K$ and $a, b \in U$ with

$$a + b \in U, \quad \lambda \cdot a \in U$$

- A line in $\mathbb{R}^2$ and $\mathbb{R}^3$ containing the origin are linear subspaces.
- A plane in $\mathbb{R}^3$ containing the origin is a linear subspace of $\mathbb{R}^3$.

9 Linear independency

A non-empty subset $U \subset V$ is called *linear independent* if a finite number of vectors in U are linear independent.

$$\lambda_1, \ldots, \lambda_m \in K, \quad a_1, \ldots, a_m \in V$$

A vector like

$$a = \lambda_1 a_1 + \ldots + \lambda_m a_m$$

is called linear combination of vectors $(a_1, \ldots, a_m)$. A linear combination is *trivial* if $\lambda_1 = \ldots = \lambda_m = 0$.

Vectors $a_1, \ldots, a_m$ are linear dependent if there is a non-trivial linear combination with

$$\lambda_1 a_1 + \ldots + \lambda_m a_m = 0$$

These vectors are linear independent if

$$\lambda_1 = 0, \ldots, \lambda_m = 0$$

Is $U \subset V$ is a non-empty subspace. The set of all linear combinations of vectors of U $(= L(U))$ is called the spanned space by $a_i \in U$.

$$L(U) = \left\{ a = \sum_i \lambda_i a_i, \lambda_i \in K, a_i \in U \right\}$$

Is $U = \{a_1, \ldots, a_m\}$:

$$L(U) = L(a_1, \ldots, a_m)$$

10 Base of a vector space

V is a vector space of K. A subspace $U \subset V$ of linear independent vectors is called basis of V if $L(U) = V$. A vector space with a finite basis is called finite-dimensional.

All bases in a finite-dimensional vector space V have the same number of vectors. This number is called dimension $(\dim V)$.

- The base of $\mathbb{R}^n$ is $\{e_1, \ldots, e_n\}$. $\dim \mathbb{R}^n = n$
- The base of $\mathbb{P}^m$ is $\{1, x, \ldots, x^m\}$. $\dim \mathbb{P}_m = m + 1$

V is a vector space with basis $B = \{v_1, \ldots, v_n\}$. Each vector $v \in V$ can be created by unique scalars $\lambda_1, \ldots, \lambda_n$.

$$v = \lambda_1 v_1 + \ldots + \lambda_n v_n$$

11 Diagonalisation

Input: A diagonalisable matrix $A \in M(n \times n)$
Output: diagonal matrix D with $B = C^{-1}DC$
Wolframalpha: Diagonalize[A]

Skriptum. page 63

The eigenvalues of a diagonal matrix are the diagonal elements. A is diagonalisable if A has n linear independent eigenvectors. C can be created by linear independent eigenvectors of A as columns in C. This structure satisfies $D = C^{-1}AC$.

12 Eigenvalues

Input: $A \in M(n \times n)$
Output: a scalar λ
Wolframalpha: *eigenvalues*[A]
Skriptum. page 59

$$\det(A - \lambda I) = 0$$

with λ as unknown variable. If you resolve the determinant, you will get a polynomial you want to know the Zero of. This polynomial is called "characteristic polynomial of A". In a polynomial of degree n you will get n solutions and therefore n eigenvalues λ. Row swapping is allowed.

λ satisfies:

$$A \cdot v = \lambda \cdot v$$

13 Eigenvectors

Input: $A \in M(n \times n)$ and eigenvalues $\sigma_{1,\dots,n}$
Output: n linear independent vectors
Wolframalpha: eigenvectors[A]
Skriptum. page 59

If $\sigma_i \neq \sigma_j \quad \forall i, j \in [1, n], i \neq j$ then linear independence is given for eigenvectors for sure.

$$(A - \lambda_i I) \cdot v_i = 0$$

Solve this equation system for v_i which will be your eigenvector.

14 Gram-Schmidt process

Input: 2 or more vectors
Output: as many vectors as given by input
Wolframalpha: Orthogonalize[$\{A, B, C\}$]
Skriptum. page 56

$$w_1 = \frac{1}{\|v_1\|} \cdot v_1$$

$$w_i = \frac{1}{\|u_i\|} \cdot u_i, \quad i = 2, \dots, n$$

$$u_i = v_i - \sum_{k=1}^{i-1} \langle v_i, w_k \rangle w_k, \quad i = 2, \dots, n$$

15 Pseudoinverse

Input: Matrix $A \in M(m \times n)$
Output: $A^{\#} \in M(n \times m)$
Wolframalpha: PseudoInverse[A]
Skriptum. page 92
More precise name: Moore-Penrose-Inverse

1. Evaluate $A^T \cdot A$
2. Evaluate $(A^T \cdot A)^{-1}$
3. Evaluate $(A^T \cdot A)^{-1} \cdot A^T = A^{\#}$

Probably the evaluation of the inverse is impossible. In this case, the pseudoinverse might be possible to evaluate using the singular value decomposition (SVD):

$$A = U\Sigma V^T \Rightarrow A^{\#} = V\Sigma^{\#}U^T$$

$\Sigma^{\#}$ can be created by inverting all singular values in D: σ_1^{-1}, σ_2^{-1}, σ_i^{-1}.

16 Singular value decomposition

Input: Matrix $A \in M(n \times n)$
Output: Matrices U, Σ, V where $A = U\Sigma V^T$
Wolframalpha: SVD[A]
Skriptum. page 68
Deutsch Singulärwertzerlegung

$$A(m \times n) = U(m \times m) \cdot \Sigma(m \times n) \cdot V(n \times n)^T$$

If A is positiv definit and symmetrical, the procedure is the same like orthogonal diagonalisation.

1. Evaluate $A^T \cdot A$

2. Evaluate die eigenvalues of $A^T \cdot A$: $\lambda_1, \lambda_2, \ldots$
3. Sort the eigenvalues by value
4. The singular values $\sigma_1, \sigma_2, \ldots$ are the squareroots of the eigenvalues $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots$
5. Evaluate the eigenvectors $v_1, v_2, \ldots$
6. Normalize the eigenvectors ($\leftarrow$ length is 1)
7. Combine the eigenvectors as column vectors $\{v_1, v_2, \ldots\} = V$
8. Create a $n \times n$ matrix and insert the σ_i as diagonals:

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & \vdots & 0 \\ 0 & 0 & \sigma_n \end{pmatrix}$$

9. Evaluate $u_i = \frac{1}{\sigma_i} \cdot A \cdot v_i$ or find other orthonormal vectors
10. Combine the vectors as column vectors $\{u_1, u_2, \ldots\} = U$
11. Evaluate V^T

U and V are not unique.

17 Gauss-Seidel Iteration

Input: Linear equation system $(A \mid b)$ and start vector x_0
Output: A vector close to the solution of the equation system
Skriptum. page 87

If no start vector is given, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is prefered.

The Gauss-Seidel-Iteration converges if

- either A is positive definit
- or for each eigenvector λ of $S^{-1}T$ it states $|\lambda| < 1$

18 Spectral radius

Input: A of an iteration algorithm
Output: a scalar
Skriptum. page 86

In the context of iteration algorithms, A is typically $S^{-1}T$.

$$\rho(A) := \max_i |\lambda_i|$$

with λ_i as eigenvalue of A.

19 Condition number

Input: regular matrix $A \in M(n \times n)$
Output: a scalar
Skriptum. page 82

$$\text{cond}(A) := \|A^{-1}\| \cdot \|A\|$$

$$\|A\|_\infty = \max \left\{ \sum_{j=1}^{n} |a_{ij}| \mid i = 1, \ldots, n \right\}$$

20 Interpolation and Approximation

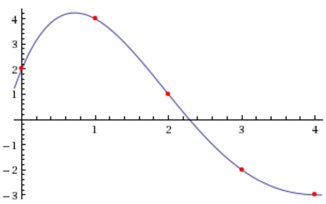

Figure 4: Interpolation

21 Cubic Spline Interpolation

Input: List of $\{x, y\}$ pairs
Output: A polynomial
Wolframalpha: BSplineCurve[$pts, SplineDegree \rightarrow 3$]

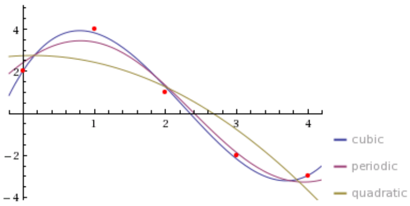

Figure 5: Approximation

Skriptum. page 115

Is S(x) a cubic spline interpolation?

- The function has to be continous. So all functions must stop at the same value at the borders.
- $f_i'(x_i) = f_{i+1}'(x_i)$ must be satisfied for all i

22 Glossary

identity matrix $a_{ii} := 1, 0$ otherwise

positiv definit A is positiv definit if *one* of the following requirements is satisfied:

$$x^T Ax > 0 \quad \forall x \in \mathbb{R}^n, x \neq 0 \tag{1}$$

$$\lambda_i > 0 \quad \forall \lambda_i \text{ of } A \tag{2}$$

$$m > 0 \quad \forall m \text{ as minor of } A \tag{3}$$

$$d_i > 0 \quad \forall d \in A_t \tag{4}$$

with d as the pivot elements of the triangular form (*without* row swapping) of A.

regular matrix $M(m\times n)$ has an inverse matrix

singular matrix $M(m\times n)$ has no inverse matrix

similar matrix $A, B \in M(n \times n)$ are similar if $C \in M(n \times n)$ in $B = C^{-1}AC$ exists; A and B have the same characteristic polynomial and the same eigenvalues

spectral radius $\rho(S^{-1}T) := \max_i |\lambda_i|$

strongly diagonal dominant

$$|a_{ii}| > \sum_{\substack{j=1\\j\neq i}}^{n} |a_{ij}| \quad \forall i = 1, \ldots, n$$

symmetrical matrix $a_{ij} = a_{ji} \quad \forall a \in M(m \times n)$

permutation An identity matrix where the same elementary row operations of the Gaussian algorithm have been applied on

triangular form $a_{ij} = 0 \quad \forall i > j$ or $\forall i < j$

quadratic matrix $A \in M(n \times m) : n = m$