Software paradigms exam 27.6.2011

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Contents

1 Exercise 1

	S	A	B	C
FIRST	$\{\underline{a}\}$	$\{\underline{a}\}$	$\{\underline{b}, \underline{c}, \underline{a}\}$	$\{\underline{c}, \varepsilon\}$
FOLLOW	$\{\$\}$	$\{\$, \underline{b}, \underline{c}, \underline{a}\}$	$\{\$, \underline{b}, \underline{c}, \underline{a}\}$	$\{\underline{a}\}$

2 Exercise 2a

	$\underline{a}$	$\underline{b}$	$\underline{c}$	$
S	$S \to A$	$S \to A$	$S \to A$	
A	$A \to AB, A \to \underline{a}$			
B	$B \to CA$	$B \to \underline{b}$	$B \to CA$	
C	$C \to \varepsilon$		$C \to \underline{c}D, C \to \underline{c}$	
D		$D \to \underline{b}$	$D \to \underline{c}$	

$(A, \underline{a})$ contains a left recursion. $(C, \underline{c})$ contains an ambiguity.

3 Exercise 2b

$$S \to IR$$
$$R \to CIR$$
$$R \to \varepsilon$$

$$I \to \underline{a}J$$
$$J \to \underline{b}J$$
$$J \to \varepsilon$$

$$C \to \underline{c}D$$
$$C \to \varepsilon$$
$$D \to \underline{c}$$
$$D \to \underline{b}$$
$$D \to \varepsilon$$

	$\underline{a}$	$\underline{b}$	$\underline{c}$	$
S	$S \to IR$			
R	$R \to CIR$		$R \to CIR$	$R \to \epsilon$
I	$I \to \underline{a}J$			
J	$J \to \varepsilon$	$J \to \underline{b}J$	$J \to \varepsilon$	$J \to \varepsilon$
C	$C \to \varepsilon$		$C \to \underline{c}D$	
D	$D \to \varepsilon$	$D \to \underline{b}$	$D \to \underline{c}$	

4 Exercise 2c

INPUT	STACK	COMMENT
$\underline{\$abcbba}$	S	$S \to IR$
$\underline{\$abcbba}$	RI	$I \to \underline{a}J$
$\underline{\$abcbba}$	$RJ\underline{a}$	$J \to \underline{b}J$
$\underline{\$abcbb}$	$RJ\underline{b}$	$J \to \underline{b}J$
$\underline{\$abcb}$	$RJ\underline{b}$	$J \to \varepsilon$
$\underline{\$abc}$	R	$R \to CIR$
$\underline{\$abc}$	RIC	$C \to \underline{c}D$
$\underline{\$abc}$	$RID\underline{c}$	$D \to \underline{b}$
$\underline{\$ab}$	$RI\underline{b}$	$I \to \underline{a}J$
$\underline{\$a}$	$RJ\underline{a}$	$J \to \varepsilon$
$\underline{\$}$	R	$R \to \varepsilon$
$\underline{\$}$		accepted

5 Exercise 3

```
factorial(x) =
  if eq?(x, 0) then eins
  else if eq?(x, eins) then eins
  else mult(factorial(minus(x, eins)), x)
```

Proof by complete induction.

1 Induction hypothesis

$$\forall \omega, \omega(\underline{x}) \leq n, \omega(\underline{x}) \in \; : I(\delta, \omega, \underline{\text{factorial}(x)}) = \omega(\underline{x})!$$

2 Induction base 0

$$\omega(\underline{x}) = 0$$

$$I(\delta, \omega, \underline{\text{if eq?(x, 0) then eins else if eq?}} \ldots)$$

$$\text{NR:}\;\; I(\delta, \omega, \underline{\text{eq?(x, 0)}}) = \text{eq?}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{0})) = \text{eq?}(\omega(\underline{x}), 0) = \mathsf{T}$$

$$I(\delta, \omega, \underline{\text{eins}}) = 1 = 0!$$

3 Induction base 1

$$\omega(\underline{x}) = 1$$

$$I(\delta, \omega, \underline{\text{if eq?(x, 0) then eins else if eq?}} \ldots)$$

$$\text{NR: } I(\delta, \omega, \underline{\text{eq?(x, 0)}}) = \text{eq?}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{0})) = \text{eq?}(\omega(\underline{x}), 0) = \mathsf{F}$$

$$I(\delta, \omega, \underline{\text{if eq?(x, 1) then eins else mult}} \ldots)$$

$$\text{NR: } I(\delta, \omega, \underline{\text{eq?(x, eins)}}) = \text{eq?}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{\text{eins}})) = \text{eq?}(\omega(\underline{x}), 1) = \mathsf{T}$$

$$I(\delta, \omega, \underline{\text{eins}} \ldots) = 1 = 1!$$

4 Induction step

$$\omega(\underline{x}) = n + 1 \qquad n \geq 2$$

$$I(\delta, \omega, \underline{\text{if eq?(x, 0) then eins else if eq?}} \ldots)$$

$$\text{NR: } I(\delta, \omega, \underline{\text{eq?(x, 0)}}) = \text{eq?}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{0})) = \text{eq?}(\omega(\underline{x}), 0) = \mathsf{F}$$

$$I(\delta, \omega, \underline{\text{if eq?(x, 1) then eins else mult}} \ldots)$$

$$\text{NR: } I(\delta, \omega, \underline{\text{eq?(x, eins)}}) = \text{eq?}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{\text{eins}})) = \text{eq?}(\omega(\underline{x}), 1) = \mathsf{F}$$

$$I(\delta, \omega, \underline{\text{mult(factorial(minus(x, eins)), x)}}) = \text{mult}(I(\delta, \omega, \underline{\text{factorial(minus(x, eins))}}), I(\delta, \omega, \underline{x}))$$

$$\text{NE: } \omega'(\underline{x}) = I(\delta, \omega, \underline{\text{minus(x, eins)}}) = \text{minus}(I(\delta, \omega, \underline{x}), I(\delta, \omega, \underline{\text{eins}}))$$

$$= \text{minus}(\omega(\underline{x}), 1) = \text{minus}(n + 1, 1) = n$$

$$\text{mult}(n!, n + 1) = (n + 1)! \qquad \text{corresponds to hypothesis}$$

6 Exercise 4a

```
odd(binStack(V, R)) :- odd(R).
odd(binStack(s(0), null)).
```

7 Exercise 4b

Tables 1 and 2.

Q	$\neg\, \text{len}(\text{binStack}(0, \text{binStack}(0, \text{binStack}(0, \text{null}))), s(s(s(0))))$
C1	$\text{add}(X, 0, X)$
C2	$\text{add}(X, s(Y), s(Z)) \vee \neg\, \text{add}(X, Y, Z)$
C3	$\text{len}(\text{null}, 0)$
C4	$\text{len}(\text{binStack}(V, R), E) \vee \neg\, \text{len}(R, S) \vee \neg\, \text{add}(s(0), S, E)$

Table 1: Rules of exercise 4b in logical notation

8 Exercise 5

$$\pi[\text{add}](\pi(\mathfrak{a}), \pi(\mathfrak{b})) = \text{add}(\mathfrak{a}, \mathfrak{b})$$

```
add(a, b) =
  if eq?(mod(a, 2), 1) then
    if eq?(mod(b, 2), 1) then
      sub(add(a, b), 1)
    else
      if gt?(a, b) then
        sub(a, b)
      else
        add(sub(b, a), 1)
  else
    if eq?(mod(b, 2), 1) then
      if gt?(a, b) then
        add(sub(a, b), 1)
      else
        sub(b, a)
    else
      add(a, b)
```

1. Sign of first parameter is positive, second parameter the same
2. Sign of first parameter is positive, second parameter is negative
3. Sign of first parameter is negative, second parameter is positive
4. Sign of first parameter is negative, second parameter the same

Case both parameters are positive:
If both parameters are positive, the modulo value of both values by 2 is 1. Therefore the very last line of the function is evaluated. For decoded values the result is $\mathfrak{a} + \mathfrak{b}$, which equals to $\pi(\mathfrak{a}) + \pi(\mathfrak{b})$ for encoded values as far as the sum of two even numbers is even (which represents a positive number).

Q	$\neg\,\text{len}(\text{binStack}(0, \text{binStack}(0, \text{binStack}(0, \text{null}))), \text{s}(\text{s}(\text{s}(0))))$
C4	$\text{len}(\text{binStack}(V, R), E) \vee \neg\,\text{len}(R, S) \vee \neg\,\text{add}(\text{s}(0), S, E)$
Θ	$\{V = 0, R = \text{binStack}(0, \text{binStack}(0, \mathtt{null})), E = \text{s}(\text{s}(\text{s}(0)))\}$
R	$\neg\,\text{len}(\text{binStack}(0, \text{binStack}(0, \text{null})), S_1) \vee \neg\,\text{add}(\text{s}(0), S_1, \text{s}(\text{s}(\text{s}(0))))$
C4	$\text{len}(\text{binStack}(V, R), E) \vee \neg\,\text{len}(R, S) \vee \neg\,\text{add}(\text{s}(0), S, E)$
Θ	$\{V_2 = 0, R_2 = \text{binStack}(0, \text{null}), E_2 = S_1\}$
R	$\neg\,\text{len}(\text{binStack}(0, \text{null}), S_2) \vee \neg\,\text{add}(\text{s}(0), S_2, S_1) \vee \neg\,\text{add}(\text{s}(0), S_1, \text{s}(\text{s}(\text{s}(0))))$
C3	$\text{len}(\text{null}, 0)$
Θ	$\{S_3 = 0\}$
R	$\neg\,\text{add}(\text{s}(0), 0, S_2) \vee \neg\,\text{add}(\text{s}(0), S_2, S_1) \vee \neg\,\text{add}(\text{s}(0), S_1, \text{s}(\text{s}(\text{s}(0))))$
C1	$\text{add}(X, 0, X)$
Θ	$\{S_2 = \text{s}(0), X = S_2\}$
R	$\neg\,\text{add}(\text{s}(0), \text{s}(0), S_1) \vee \neg\,\text{add}(\text{s}(0), S_1, \text{s}(\text{s}(\text{s}(0))))$
C2	$\text{add}(X, \text{s}(Y), \text{s}(Z)) \vee \neg\,\text{add}(X, Y, Z)$
Θ	$\{X_2 = \text{s}(0), Y_2 = 0, Z_2 = A, S_1 = \text{s}(A)\}$
R	$\neg\,\text{add}(\text{s}(0), 0, A) \vee \neg\,\text{add}(\text{s}(0), \text{s}(A), \text{s}(\text{s}(\text{s}(0))))$
C1	$\text{add}(X, 0, X)$
Θ	$\{X_3 = \text{s}(0), A = X_3\}$
R	$\neg\,\text{add}(\text{s}(0), \text{s}(\text{s}(0)), \text{s}(\text{s}(\text{s}(0))))$
C2	$\text{add}(X, \text{s}(Y), \text{s}(Z)) \vee \neg\,\text{add}(X, Y, Z)$
Θ	$\{X_4 = \text{s}(0), Y_3 = \text{s}(0), Z_3 = \text{s}(\text{s}(0))\}$
R	$\neg\,\text{add}(\text{s}(0), \text{s}(0), \text{s}(\text{s}(0)))$
C2	$\text{add}(X, \text{s}(Y), \text{s}(Z)) \vee \neg\,\text{add}(X, Y, Z)$
Θ	$\{X_5 = \text{s}(0), Y_4 = 0, Z_4 = \text{s}(0)\}$
R	$\neg\,\text{add}(\text{s}(0), 0, \text{s}(0))$
C1	$\text{add}(X, 0, X)$
Θ	$\{X_6 = \text{s}(0)\}$
R	empty, query got successfully derivated.

Table 2: Derivation of the query in exercise 4b